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A NOTE ON TRANSFORMATION IN SAMPLING

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SUMMARY

It has been shown that bias and mean square error of the transformed ratio estimator of the population mean are minimised at the same value of increment given to each value of the auxiliary variate.

Keywords : Ratio estimator; Product estimator; Bias; Mean square error; Modified ratio estimator; Modified product estimator.

Mohanty and Das [2] introduced the use of transformation as a tool for reduction of M.S.E. (Mean Square Error) and bias of the ratio estimator of the population mean simultaneously. Related works in this area are available in the papers of Kulkarni [1], Sisodia and Dwibedi [3]. In the paper of Mohanty and Das [2], it has been mentioned that replacing X (the auxiliary variate) by $[X + (\alpha/\beta)]$ the bias of the ratio estimator becomes zero where $Y = \alpha + \beta X$ is the regression line of Y on X in the population. The object of this note is to point out that if each value of the concomitant variable X is changed by an amount (α/β), not only the bias of the ratio estimator of the population mean becomes zero but also its m.s.e. reduces to minimum value. Moreover, a study on the use of transformation on the product estimator reveals that an increment to each value of the auxiliary variable by an amount $-[(\bar{Y} + \beta\bar{X})/\beta]$ reduces the value of the mean square error of the product estimator of the population mean to a minimum. On the other hand, the bias of the above mentioned product estimator can be reduced by changing each value of the auxiliary variable X by a sufficiently large amount, If the usual ratio estimator for population mean is

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \cdot \bar{X}, \tag{1}$$

then the modified ratio estimator is

$$\hat{\bar{Y}}_{MR} = \frac{\bar{y}}{\bar{x}'} \cdot \bar{X}', \tag{2}$$

where $\bar{X}' = \bar{X} + B$, $\bar{x}' = \bar{x} + B$.
Similarly, if the usual product estimator of the population mean is

$$\hat{\bar{Y}}_P = \frac{\bar{y} \cdot \bar{x}}{\bar{X}}, \tag{3}$$

then the modified product estimator is

$$\hat{\bar{Y}}_{MP} = \frac{\bar{y} \cdot \bar{x}'}{\bar{X}'}, \tag{4}$$

where $\bar{X}'$, $\bar{x}'$ are defined as before.

The expression for bias and mean square error are as follows :

$$\text{Bias}\ (\hat{\bar{Y}}_R) = C_2\,(\bar{Y}/\bar{X}) \cdot (1/\bar{X}) - C_4\,(1/\bar{X}) \tag{5}$$

$$\text{M.E.S.}\ (\hat{\bar{Y}}_R) = C_1 + C_2\,(\bar{Y}/\bar{X})^2 - C_3\,(\bar{Y}/\bar{X}) \tag{6}$$

$$\text{Bias}\ (\hat{\bar{Y}}_P) = C_3\,(1/\bar{X}) \tag{7}$$

$$\text{M.S.E.}\ (\hat{\bar{Y}}_P) = C_1 + C_2\,(\bar{Y}/\bar{X})^2 + C_3\,(\bar{Y}/\bar{X}) \tag{8}$$

where $C_1 = \theta\, S_Y^2$, $\quad C_4 = C_{3/2}$,

$C_2 = \theta\, S_X^2$,

$C_3 = 2\,\theta\, S_{XY}$, $\quad \theta = \dfrac{N - n}{Nn}$.

The bias and mean square error of $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$ are obtained by replacing $\bar{X}$ by $(\bar{X} + B)$ in the equation (5), (6), (7), (8) respectively.

$$\text{Bias } (\hat{\bar{Y}}_{MR}) = C_2 \, (\bar{Y}/(\bar{X} + B)) \, (1/(\bar{X} + B)) - C_4 \, (1/(\bar{X} + B)) \tag{9}$$

$$\text{M.S.E. } (\hat{\bar{Y}}_{MR}) = C_1 + C_2 \left(\frac{\bar{Y}}{\bar{X} + B} \right)^2 - C_3' \left(\frac{\bar{Y}}{\bar{X} + B} \right) \tag{10}$$

$$\text{Bias} = (\hat{\bar{Y}}_{MP}) = C_3 \left(\frac{1}{\bar{X} + B} \right) \tag{11}$$

$$\text{M.S.E. } (\hat{\bar{Y}}_{MP}) = C_1 + C_2 \left(\frac{\bar{Y}}{\bar{X} + B} \right)^2 + C_3 \left(\frac{\bar{Y}}{\bar{X} + B} \right) \tag{12}$$

The solution of the equation Bias $(\hat{Y}_{MR}) = 0$ is $B = (\alpha/\beta)$, where $\alpha = \bar{Y} - \beta \bar{X}$ and $\beta = C_4/C_2 = \rho S_Y/S_X = C_3/2C_2 =$ regression coefficient.

Also, minimum M.S.E. $(\hat{\bar{Y}}_{MR})$ is obtained for $B = (\alpha/\beta)$, $B > 0$. This can be proved as follows : Differentiating M.S.E. $(\hat{\bar{Y}}_{MR})$ presented at (10) with respect to B and equating the differential to zero, we solve for B. This solution of B gives the value of B for which the M S.E. is minimum. Thus

$$\frac{d\,(\text{M.S.E.})}{dB} = 0$$

$$\Rightarrow \frac{-2 \, C_2 \, \bar{Y}^2}{(\bar{X} + B)^3} + \frac{C_3 \, \bar{Y}}{(\bar{X} + B)^2} = 0$$

$$\Rightarrow \frac{\bar{Y}}{(\bar{X} + B)^3} \, [C_3 \, (\bar{X} + B) - 2C_2 \bar{Y}] = 0$$

$$\therefore \; B = \frac{2 \, C_2 \, \bar{Y}}{C_3} - \bar{X}$$

$$= 1/\beta \, \bar{Y} - \bar{X} \qquad \text{as } \beta = \frac{C_3}{2C_2}$$

$$= 1/\beta \, (\bar{Y} - \beta \bar{X})$$

$$= \alpha/\beta \qquad \text{where } \alpha = \bar{Y} - \beta \, \bar{X}.$$

Hence the transformation $Z = X + \alpha/\beta$ makes the modified ratio estimate $[\bar{y}(\bar{x} + B)]$. $(\bar{X} + B)$ unbiased and also minimises M.S.E. In case of product estimator of the population mean, minimum M.S.E. $(\hat{\bar{Y}}_{MP})$ is obtained for $B = -[(\bar{Y} + \bar{X}\beta)/\beta]$, $B > 0$. In fact, Min M.S.E. $(\hat{\bar{Y}}_{MR}) = C_1 + C_2\beta^2 - C_3\beta$ and Min M.S.E. $(\hat{\bar{Y}}_{MP}) = C_1 + C_2\beta^2 + C_3\beta$:

$$B = \alpha/\beta \qquad\qquad B = -\left(\frac{\bar{Y} + \bar{X}\beta}{\beta}\right)$$

Suppose a change B is effected on each value of the variables X and Y respectively; then, at the same value $B = \alpha/(\beta - 1)$, minimum M.S.E. $(\hat{\bar{Y}}_{MR})$ and the solution of the equation Bias $(\hat{\bar{Y}}_{MR}) = 0$, $B \ngtr 0$, are obtained, where,

$$\hat{\bar{Y}}_{MR} = \frac{\bar{y} + B}{\bar{x} + B}(\bar{X} + B) \tag{13}$$

The proof is as follows :

$$\hat{\bar{Y}}_{MR} = \frac{\bar{y} + B}{\bar{x} + B}(\bar{X} + B),$$

$$\text{M.S.E. } (\hat{\bar{Y}}_{MR}) = C_1 + C_2\left(\frac{\bar{Y} + B}{\bar{X} + B}\right)^2 - C_3\left(\frac{\bar{Y} + B}{\bar{X} + B}\right).$$

Differentiating this M.S.E. with respect to B and equating the differential to zero, we solve for B. This solution of B gives the value of B for which the M.S.E. is minimum. Thus

$$\frac{d\,(\text{M.S.E.})}{dB} = 0$$

$$\Rightarrow \frac{d}{dB}\left[C_1 + C_2\left(\frac{\bar{Y} + B}{\bar{X} + B}\right)^2 - C_3\left(\frac{\bar{Y} + B}{\bar{X} + B}\right)\right] = 0$$

$$\Rightarrow C_2\left[-\frac{2(\bar{Y} + B)^2}{(\bar{X} + B)^3} + 2\frac{(\bar{Y} + B)}{(\bar{X} + B)^2}\right] - C_3\left[1/(\bar{X} + B) - \frac{\bar{Y} + B}{(\bar{X} + B)^2}\right] = 0$$

$$\Rightarrow \frac{\bar{Y} + B}{\bar{X} + B}[-2C_2(\bar{Y} + B) + 2C_2(\bar{X} + B)] - [C_3(\bar{X} + B) - C_3(\bar{Y} + B)] = 0$$

$$\Rightarrow (\bar{Y} + B)\, 2\, C_2 (\bar{X} - \bar{Y}) = (\bar{X} + B)\, C_3 (\bar{X} - \bar{Y})$$

$$\therefore\ B = \frac{C_3 \bar{X} - 2\, C_2 \bar{Y}}{2\, C_2 - C_3} = \frac{\bar{Y} - \beta \bar{X}}{\beta - 1} = \alpha/(\beta - 1).$$

It makers the ratio estimate (13) unbiased and also minimises M.S.E. In case of the product estimator of the population mean, minimum M.S.E. ($\bar{Y}_{MP}$) is obtained for $B = -(\bar{Y} + \bar{X}\beta)/(\beta + 1)$, $B > 0$, where,

$$\hat{\bar{Y}}_{MP} = \frac{(\bar{y} + B) \cdot (\bar{x} + B)}{(\bar{X} + B)}.$$

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